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ON HYDROMAGNETIC TRIPLY DIFFUSIVE CONVECTION IN COUPLE-STRESS FLUID THROUGH POROUS MEDIUM

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ABSTRACT
The triply diffusive convection in couple-stress fluid is mathematically investigated in the presence of uniform vertical magnetic field through porous medium. Using linearized stability theory and normal mode analysis, the dispersion relation is obtained. The magnetic field, couple-stress parameter and stable solute gradients are found to have stabilizing effects, whereas medium permeability has a destabilizing effect for stationary convection. Further, solute gradients, couple-stress parameter and magnetic field are found to introduce oscillatory modes, which were non-existent in their absence. The sufficient conditions for the non-existence of over stability are also obtained.

KEYWORDS: Triply diffusive convection; Solute gradients; Couple-stress fluid; Uniform vertical magnetic field; Porous medium

1. INTRODUCTION
The thermal instability of a fluid layer heated from below plays an important role in geophysics, atmospheric physics, oceanography etc., and has been investigated by many authors, e.g. Be'nard (1900), Rayleigh (1916), Jeffreys (1926). A detailed account of the theoretical and experimental studies of so called Be'nard convection in Newtonian fluids has been given by Chandrasekhar (1981).

Thermal convection is the most convective instability when crystals are produced from single element like silicon. However, gallium arsenide and other semi-conductors which require crystals made from compounds of elements are beginning to take on a prominent position in modern technologies. Hence, at present there is strong industrial demand for understanding the additional effects that can occur in the solidification of a mixture, which is not possible in one component system. The problem of thermohaline convection in a fluid layer heated from below and subjected to a stable salinity gradient has been considered by Veronis (1965). The buoyancy force can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. Double-diffusive convection problems arise in oceanography (salt fingers occur in the ocean when hot saline water overlies cooler fresher water which believed to play an important role in the mixing of properties in several regions of the ocean), limnology and engineering. There are many situations in which double-diffusive convection is involved like migration of moisture in fibrous insulation, underground disposal of nuclear waste, groundwater, bio/chemical contaminants transport in environment, magmas, high quality crystal production and production of pure medication.

Although the subject of double-diffusive convection is still an area of active research, however, there are many fluid dynamical systems occurring in industrial applications and nature involve three or more stratifying agencies having different molecular diffusivities. More complex systems can be obtained in magmas and molten metals (Jakeman and Hurle (1972)). This has prompted researchers to study convective instability in triple diffusive fluid systems both theoretically and experimentally (Turner (1985), Terrones and Pearlstein (1989), Pearlstein et al. (1989), Lopez et al. (1990)). The effects of cross-diffusion on the horizontally unbounded triply cross-diffusion fluid layer have been investigated by Terrones (1993). Straughan and Walker (1997) have analyzed various aspects of penetrative convection in a triply diffusive fluid layer, while multicomponent convection-diffusion with internal heating or cooling in a fluid layer has been discussed by Straughan and Tracery (1999).

The previous studies on triple diffusive convection in a fluid layer are dealt with only Newtonian fluid theory. As propounded earlier, many fluid dynamical systems such as molten polymers, geothermally heated lakes, salt solutions, slurries, magmas and their laboratory models, synthesis of chemical compounds usually involve more than two diffusing components and can be well characterized by couple stress fluid theory rather than Newtonian theory. The couple-stress fluid theory represents the simplest generalization of the classical viscous fluid theory that allows for polar effects and whose microstructure is mechanically significant in fluids. For such a special kind of non-Newtonian fluids, the constitutive equations are given by Stokes (1966) which allows the sustenance of couple-stresses in addition to usual stresses.

Many of the flow problems in fluids with couple-stresses, discussed by Stokes, indicate some possible experiments, which could be used for determining the material constants, and the results are found to differ from those of Newtonian fluid. Couple-stresses are found to appear in noticeable magnitudes in polymer solutions for force and couple-stresses. This theory is developed in an effort to examine the simplest generalization of the classical theory, which would allow polar effects. The constitutive equations proposed by Stokes (1966) are:

$$T_{(ij)} = (-p + \lambda D_{kk})\delta_{ij} + 2\mu D_{ij}, \quad T_{[ij]} = -2\eta \vec{W}_{ij.kk} - \frac{\rho}{2}\vec{\varepsilon}_{ijs} G_s,$$

and $$M_{ij} = 4\eta \vec{\omega}_{j,i} + 4\eta' \vec{\omega}_{i,j},$$

where $$D_{ij} = \frac{1}{2}(V_{i,j} + V_{j,i}), \quad \vec{W}_{ij} = -\frac{1}{2}(V_{i,j} - V_{j,i})$$

and $$\vec{\omega}_i = \frac{1}{2}\vec{\varepsilon}_{ijk} V_{k,j} \quad .$$

Here T_{ij}, $T_{(ij)}$, $T_{[ij]}$, M_{ij}, D_{ij}, $\vec{W}_{i,j}$, $\vec{\omega}_i$, G_s, $\vec{\varepsilon}_{ijk}$, V, ρ and λ, μ, η, η', are stress tensor, symmetric part of T_{ij}, anti-symmetric part of T_{ij}, the couple-stress tensor, deformation tensor, the vorticity tensor, the vorticity vector, body couple, the alternating unit tensor, velocity field, the density and material constants respectively. The dimensions of λ and μ are those of viscosity whereas the dimensions of η and η' are those of momentum.

Goel et al. (1999) have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Kumar et al. (2004) have considered the thermal instability of a layer of a couple-stress fluid acted on by a uniform rotation, and have found that for stationary convection, the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects. Thermosolutal convection in a couple-stress fluid in presence of magnetic field and rotation, separately, has been investigated by Kumar and Singh (2008, 2009). Kumar and Kumar (2010) studied the problem on a couple-stress fluid heated from below in hydromagnetics and found that magnetic field has a stabilizing effect on the system. The effect of magnetic field on an incompressible (Kuvshiniski-type) viscoelastic rotating fluid heated from below in porous medium is considered by Kumar and Kumar (2013) and found that magnetic field play stabilizing role in certain conditions.

Recently, interest in viscoelastic flows through porous media has grown considerably, due largely to the demands of such diverse fields as biorheology, geophysics, chemical, and petroleum industries. Keeping in mind the importance in various fields particularly in the soil sciences, ground water-hydrology, geophysics, astrophysics and bio-mechanics, the triply diffusive convection in couple-stress fluid through porous medium in the presence of magnetic field has been considered in the present paper.

2. FORMULATION OF THE PROBLEM

Here we consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated and soluted from below so that, the temperature T and solute concentrations $C^{(1)}$ and $C^{(2)}$ at the bottom surface $z = 0$ are $T_0, C_0^{(1)}$ and $C_0^{(2)}$; and at the upper surface $z = d$ are $T_d, C_d^{(1)}$ and $C_d^{(2)}$ respectively, and a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and uniform solute gradients $\beta'\left(=\left|\frac{dC^{(1)}}{dz}\right|\right)$ and $\beta''\left(=\left|\frac{dC^{(2)}}{dz}\right|\right)$ are maintained. The gravity field $\vec{g}(0,0,-g)$ and a uniform vertical magnetic field $\vec{H}(0,0,H)$ pervade the system.

When fluid flows through a porous medium, the gross effect is represented by Darcy's law, the equations of motion and continuity for couple-stress fluid through porous medium become

$$\frac{1}{\varepsilon}\left[\frac{\partial\vec{v}}{\partial t}+\frac{1}{\varepsilon}(\vec{v}.\nabla)\vec{v}\right]=-\frac{1}{\rho_0}\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\vec{v}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}\,, \quad (1)$$

$$\nabla.\vec{v}=0\,, \quad (2)$$

where $\vec{v}$ is the filter velocity, ε is medium porosity, k_1 is the medium permeability and $\nu(=\mu/\rho)$ the kinematic viscosity and μ' the couple-stress viscosity. The fluid velocity $\vec{q}$ and the Darcian (filter) velocity $\vec{v}$ are connected by the relation $\vec{q}=\vec{v}/\varepsilon$. A porous medium of very low permeability allows us to use the Darcy's model. For a medium of very large stable particle suspension, the permeability tends to be small justifying the use of Darcy's model. This is because the viscous drag force is negligibly small in comparison with Darcy's resistance due to the large particle suspension.

When the fluid flows through a porous medium, the equation of heat conduction is

$$(\rho c_f\varepsilon+\rho_s c_s(1-\varepsilon))\frac{\partial T}{\partial t}+\rho c_f(\vec{v}.\nabla)T=\kappa\nabla^2 T \quad (3)$$

and analogous solute concentration equations are

$$(\rho c_f{}'\varepsilon+\rho_s c_s{}'(1-\varepsilon))\frac{\partial C^{(1)}}{\partial t}+\rho c_f{}'(\vec{v}.\nabla)C^{(1)}=\kappa'\nabla^2 C^{(1)} \quad (4)$$

$$(\rho c_f{}''\varepsilon+\rho_s c_s{}''(1-\varepsilon))\frac{\partial C^{(2)}}{\partial t}+\rho c_f{}''(\vec{v}.\nabla)C^{(2)}=\kappa''\nabla^2 C^{(2)}\,, \quad (5)$$

The Maxwell's equations yield

$$\varepsilon\frac{d\vec{H}}{dt} = (\vec{H}.\nabla)\vec{v}+\varepsilon\eta\nabla^2\vec{H}\,, \quad (6)$$

$$\nabla\,.\,\vec{H}=0. \quad (7)$$

where $\frac{d}{dt}\equiv\frac{\partial}{\partial t}+\varepsilon^{-1}\vec{v}.\nabla$. Since density variations are mainly due to variations in temperature and solute concentrations, the equation of state for the fluid is given by

$$\rho=\rho_0[1-\alpha(T-T_a)+\alpha'(C-C_a^{(1)})+\alpha''(C-C_a^{(2)})], \quad (8)$$

where $\rho,\rho_0,t,\nu,\kappa,\kappa',\kappa'',\alpha,\alpha'$ and α'' are the fluid density, reference density, time, the kinematic viscosity, the thermal diffusivity, the solute diffusivities, thermal and solvent coefficients of expansion respectively. T_a is the average temperature given by $T_a=\frac{T_0+T_d}{2}$ where T_0 and T_d are the constant average temperatures of the lower and upper surfaces of the layer and $C_a^{(1)}$, $C_a^{(2)}$ are the average concentrations given by $C_a^{(1)}=\frac{C_0^{(1)}+C_d^{(1)}}{2}$ and $C_a^{(2)}=\frac{C_0^{(2)}+C_d^{(2)}}{2}$, where $C_0^{(1)}$, $C_d^{(1)}$ and $C_0^{(2)}, C_d^{(2)}$ are constant average concentrations of the lower and upper surfaces of the layer. Here $E=\varepsilon+(1-\varepsilon)\frac{\rho_s c_s}{\rho c_f}$ is a constant, E' and E'' are analogous to E but corresponding to solute rather than heat. ρ, c_f ; ρ_s, c_s stand for density and heat capacity of fluid and solid matrix, respectively.

3. BASIC STATE AND PERTURBATION EQUATIONS

The basic state was assumed to be quiescent and is given by

$$\vec{v}=(0,0,0),\ \vec{H}_b=(0,0,H),\ T=T_b(z),\ \ p=p_b(z),\ C^{(1)}=C_b^{(1)}(z),\ C^{(2)}=C_b^{(2)}(z),\ \rho=\rho_b(z),$$

$$T_b(z)=T_a-\beta z,\ C_b^{(1)}(z)=C_a^{(1)}-\beta' z,\ \ C_b^{(2)}(z)=C_a^{(2)}-\beta'' z \text{ with}$$

$$\rho_b=\rho_0\,[1-\alpha(T_b-T_a)+\alpha'(C_b^{(1)}-C_a^{(1)})+\alpha''(C_b^{(2)}-C_a^{(2)})]. \quad (9)$$

To use linearized stability theory and normal mode technique, we assume small perturbations on the basic state solution. Let $\vec{v}(u,v,w)=0+\vec{v}(u',v',w')$, $\rho=\rho_b+\rho'$, $\vec{H}=H_b+H'(h_x,h_y,h_z)$, $p=p_b+p'$, $T=T_b+T'$, $C^{(1)}=C_b^{(1)}+C^{(1)\prime}$ and $C^{(2)}=C_b^{(2)}+C^{(2)\prime}$ denote, respectively the perturbations in the fluid velocity, density, pressure, temperature and concentrations. The change in density ρ' caused mainly by the perturbations in temperature and concentrations is given by

$$\rho'=-\rho_0\,[\,\alpha T'-\alpha' C^{(1)\prime}-\alpha'' C^{(2)\prime}\,]. \quad (10)$$

Then the linearized hydromagnetic perturbation equations are

$$\frac{1}{\varepsilon}\frac{\partial \vec{v}'}{\partial t}=-\frac{1}{\rho_0}\nabla p'+g\left(\alpha T'-\alpha' C^{(1)\prime}-\alpha'' C^{(2)\prime}\right)+\frac{\mu_e}{4\pi\rho_0}(\nabla\times H')\times\vec{H}-\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\vec{v}', \quad (11)$$

$$\nabla.\vec{v}'=0, \quad (12)$$

$$E\frac{\partial T'}{\partial t}=\beta w+\kappa\nabla^2 T', \quad (13)$$

$$E'\frac{\partial C^{(1)\prime}}{\partial t}=\beta' w+\kappa'\nabla^2 C^{(1)\prime}, \quad (14)$$

$$E''\frac{\partial C^{(2)\prime}}{\partial t}=\beta'' w+\kappa''\nabla^2 C^{(2)\prime}, \quad (15)$$

$$\varepsilon\frac{dH'}{dt}=(H.\nabla)\vec{v}+\varepsilon\eta\nabla^2 H', \quad (16)$$

$$\nabla\,.\,H'=0. \quad (17)$$

Analyzing the perturbations into normal modes, we assume that the perturbation quantities are of the form

$$\left[w,T',C^{(1)\prime},C^{(2)\prime},h_z\right]=\left[W(z),\Theta(z),\Gamma(z),\Upsilon(z),K(z)\right]\exp\{ik_x x+ik_y y+nt\}, \quad (18)$$

where k_x and k_y are the wave numbers in x and y directions respectively, $k=(k_x^2+k_y^2)^{1/2}$ is the resultant wave number of propagation and n is the frequency of any arbitrary disturbance which is, in general, a complex constant. Using equation (18), equations (11) to (17) in non-dimensional form become

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}-\frac{F}{P_l}\left(D^2-a^2\right)\right](D^2-a^2)W+\frac{ga^2d^2}{\nu}(\alpha\Theta-\alpha'\Gamma-\alpha''\Upsilon)-\frac{\mu_e Hd}{4\pi\rho_0\nu}\left(D^2-a^2\right)DK=0, \quad (19)$$

$$\left(D^2-a^2-E\sigma p_1\right)\Theta=-\frac{\beta d^2}{\kappa}W, \quad (20)$$

$$\left(D^2-a^2-E'\sigma q_1\right)\Gamma=-\frac{\beta' d^2}{\kappa'}W, \quad (21)$$

$$\left(D^2-a^2-E''\sigma q_2\right)\Upsilon=-\frac{\beta'' d^2}{\kappa''}W, \quad (22)$$

$$\left(D^2-a^2-\sigma p_2\right)K=-\frac{Hd}{\varepsilon\eta}DZ. \quad (23)$$

Here we have put $a=kd$, $\sigma=\frac{nd^2}{\nu}$, $p_1=\frac{\nu}{\kappa}$, $p_2=\frac{\nu}{\eta}$, $q_1=\frac{\nu}{\kappa'}$, $q_2=\frac{\nu}{\kappa''}$, $F=\frac{\mu'}{\rho_0 d^2\nu}$, $P_l=\frac{k_1}{d^2}$ and

$D^* = dD$[(*) is dropped for convenience].

Now considering the case where both boundaries are free as well as perfect conductors of both heat and solute concentrations, while the adjoining medium is perfectly conducting. The case of two boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which equations (19)-(23) must be solved are

$W = 0,\ D^2W = 0,\ \Theta = 0,\ \Gamma = 0,\ \Psi = 0$ at $z = 0$ and $z = 1$.

$K = 0$ on a perfectly conducting boundary and h_x, h_y, h_z are continuous with an external vacuum field on a non-conducting boundary. (24)

The case of two free boundaries, though a little artificial, is the most appropriate for stellar atmospheres (Spiegel, 1965). Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{25}$$

where W_0 is constant. On eliminating various physical parameters from equations (19)-(23) and substituting the proper solution (25) in the resultant equation, we obtain the final dispersion relation as

$$R_1 = \left(\frac{1+\omega}{\omega}\right)\left[\left\{\frac{i\sigma_1}{\varepsilon} + \frac{1}{P} + \frac{F_1}{P}(1+\omega)\right\}(1+\omega+iEp_1\sigma_1) + Q_1\left[\frac{(1+\omega)(1+\omega+iEp_1\sigma_1)}{\omega(1+\omega+ip_2\sigma_1)}\right]\right.$$
$$\left. + S_1\left(\frac{1+\omega+i\sigma_1 Ep_1}{1+\omega+i\sigma_1 E' q_1}\right) + S_2\left(\frac{1+\omega+i\sigma_1 Ep_1}{1+\omega+iE''\sigma_1 q_2}\right)\right]. \tag{26}$$

Here, $R = \dfrac{g\alpha\beta d^4}{\nu\kappa\pi^4}$, $S_1 = \dfrac{g\alpha'\beta' d^4}{\nu\kappa'\pi^4}$, $F_1 = \dfrac{F}{\pi^2}$, $i\sigma_1 = \dfrac{\sigma}{\pi^2}$, $\omega = \dfrac{a^2}{\pi^2}$, $P = \pi^2 P_l$, $S_2 = \dfrac{g\alpha''\beta'' d^4}{\nu\kappa''\pi^4}$ and $Q_1 = \dfrac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon\pi^2}$.

4. STATIONARY CONVECTION

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (26) reduces to

$$R_1 = \frac{(1+\omega)^2}{\omega P} + \frac{(1+\omega)^3 F_1}{\omega P} + Q_1\frac{(1+\omega)}{\omega} + S_1 + S_2, \tag{27}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number ω and the parameters Q_1, S_1, S_2, F_1 and P.

To study the effect of solute gradients, magnetic field, couple-stress viscosity and medium permeability, we examine the nature of $\dfrac{dR_1}{dS_1}, \dfrac{dR_1}{dS_2}, \dfrac{dR_1}{dQ_1}, \dfrac{dR_1}{dF_1}$ and $\dfrac{dR_1}{dP}$ analytically.

From equation (27), we have

$$\frac{dR_1}{dS_1} = 1 \text{ and } \frac{dR_1}{dS_2} = 1, \tag{28}$$

which show that solute gradients have stabilizing effect on the triple diffusive convection in couple-stress fluid.

Equation (27) also yields

$$\frac{dR_1}{dQ_1} = \frac{(1+\omega)}{\omega}, \tag{29}$$

which show that magnetic field has stabilizing effect on the triple diffusive convection in couple-stress fluid.

Also equation (27) gives

$$\frac{dR_1}{dF_1} = \frac{(1+\omega)^3}{\omega P}, \quad (30)$$

which shows that couple-stress parameter has stabilizing effect.

From equation (27), we have

$$\frac{dR_1}{dP} = -\frac{(1+\omega)^2}{\omega P^2}\left[1+F_1(1+\omega)\right], \quad (31)$$

which shows that medium permeability has a destabilizing effect on the system.

5. SOME IMPORTANT THEOREMS

Theorem 1: The system is stable or unstable.

Proof: Multiplying equation (19) by W*, the complex conjugate of W, integrating over the range of z, and making use of equations (20)-(23) together with boundary conditions (24), we get

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right]I_1 - \frac{g\alpha\kappa a^2}{\nu\beta}[I_2+\sigma^* E p_1 I_3] + \frac{g\alpha'\kappa' a^2}{\nu\beta'}[I_4+\sigma^* E' q_1 I_5]$$

$$+\frac{g\alpha''\kappa'' a^2}{\nu\beta''}[I_6+\sigma^* E'' q_2 I_7] + \frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}[I_8+\sigma^* p_2 I_9] + \frac{F}{P_l}I_{10} = 0. \quad (32)$$

Here $I_1 = \int\left(|DW|^2 + a^2|W|^2\right)dz$, $I_2 = \int\left(|D\Theta|^2 + a^2|\Theta|^2\right)dz$, $I_3 = \int|\Theta|^2\,dz$,
$I_4 = \int\left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz$, $I_5 = \int|\Gamma|^2\,dz$, $I_6 = \int\left(|D\psi|^2 + a^2|\psi|^2\right)dz$, $I_7 = \int|\psi|^2\,dz$,
$I_8 = \int\left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz$, $I_9 = \int\left(|DK|^2 + a^2|K|^2\right)dz$,

$$I_{10} = \int\left(|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right)dz, \quad (33)$$

and σ^* is the complex conjugate of σ. The integrals $I_1 - I_{10}$ are all positive definite. Substituting $\sigma = \sigma_r + i\sigma_i$ in equation (32), where σ_r and σ_i are real and then equating the real and imaginary parts, we get

$$\sigma_r\left[\frac{I_1}{\varepsilon} - \left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)Ep_1I_3 + \left(\frac{g\alpha'\kappa' a^2}{\nu\beta'}\right)E'q_1I_5 + \left(\frac{g\alpha''\kappa'' a^2}{\nu\beta''}\right)E''q_2I_7 + \frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}p_2I_9\right] =$$

$$-\left[\frac{I_1}{P_l} - \frac{g\alpha\kappa a^2}{\nu\beta}I_2 + \frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4 + \frac{g\alpha''\kappa'' a^2}{\nu\beta''}I_6 + \frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}I_8 + \frac{F}{P_l}I_{10}\right] \quad (34)$$

and
$$\sigma_i\left[\frac{I_1}{\varepsilon} + \left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)Ep_1I_3 - \left(\frac{g\alpha'\kappa' a^2}{\nu\beta'}\right)E'q_1I_5 - \left(\frac{g\alpha''\kappa'' a^2}{\nu\beta''}\right)E''q_2I_7 - \frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}p_2I_9\right] = 0. \quad (35)$$

It is evident from equation (34) that σ_r is positive or negative. The system is, therefore, stable or unstable.

Theorem 2: The modes may be oscillatory or non-oscillatory in contrast to the case of no magnetic field and in the absence of stable solute gradients and couple-stress parameter where modes are non-oscillatory.

Proof: Equation (35) yields that σ_i may be zero or non-zero, which means that the modes may be non-oscillatory or oscillatory. In the absence of solute gradient, couple-stress parameter and magnetic field, equation (35) reduces to

$$\sigma_i\left[\frac{I_1}{\varepsilon}+\left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)Ep_1I_3\right]=0,$$
(36)

and the terms in brackets are positive definite. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is valid for the couple-stress fluid for a porous medium, in the absence of stable solute gradient, couple-stress parameter and magnetic field. The oscillatory modes are introduced due to the presence of stable solute gradient, couple-stress parameter and magnetic field, which were non-existent in their absence.

Theorem 3: The system is stable for $\frac{g\alpha\kappa P_l}{\nu\beta F}\le\frac{27\pi^4}{4}$ and under the condition $\frac{g\alpha\kappa P_l}{\nu\beta F}>\frac{27\pi^4}{4}$, the system becomes unstable.

Proof: From equation (35), it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If $\sigma_i \neq 0$, then equation (35) gives

$$\frac{I_1}{\varepsilon}=-\left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)Ep_1I_3+\left(\frac{g\alpha'\kappa' a}{\nu\beta'}\right)E'q_1I_5+\left(\frac{g\alpha''\kappa'' a^2}{\nu\beta''}\right)E''q_2I_7+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}p_2I_9, \tag{37}$$

Substituting in equation (34), we have

$$\frac{2\sigma_r}{\varepsilon}I_1+\frac{I_1}{P_l}+\frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4+\frac{g\alpha''\kappa'' a^2}{\nu\beta''}I_6+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}I_8+\frac{F}{P_l}I_{10}=\frac{g\alpha\kappa a^2}{\nu\beta}I_2. \tag{38}$$

Equation (38) on using Rayleigh-Ritz inequality gives

$$\frac{(\pi^2+a^2)^3}{a^2}\int_0^1|W|^2dz+\frac{(\pi^2+a^2)}{a^2}\frac{P_l}{F}\left\{\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}I_8+\frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4+\frac{g\alpha''\kappa'' a^2}{\nu\beta''}I_6+\frac{I_1}{P_l}+\frac{2\sigma_rI_1}{\varepsilon}\right\}$$
$$\le\frac{g\alpha\kappa P_l}{\nu\beta F}\int_0^1|W|^2dz. \tag{39}$$

Therefore, It follows from equation (39) that

$$\left[\frac{27\pi^4}{4}-\frac{g\alpha\kappa P_l}{\nu\beta F}\right]\int_0^1|W|^2dz+\left(\frac{\pi^2+a^2}{a^2}\right)\frac{P_l}{F}\left\{\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}I_8+\frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4+\frac{g\alpha''\kappa'' a^2}{\nu\beta''}I_6+\frac{I_1}{P_l}+\frac{2\sigma_rI_1}{\varepsilon}\right\}$$
$$\le 0. \tag{40}$$

Since minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ with respect to a^2 is $\frac{27\pi^4}{4}$. Now, let $\sigma_r \ge 0$, we necessary have from (40) that

$$\frac{g\alpha\kappa P_l}{\nu\beta F}>\frac{27\pi^4}{4}, \tag{41}$$

Hence, if $\frac{g\alpha\kappa P_l}{\nu\beta F}\le\frac{27\pi^4}{4}$, (42)

then $\sigma_r < 0$. Therefore, the system is stable. Therefore, under condition (42), the system is stable and under condition (41) the system becomes unstable.

Theorem 4: The sufficient conditions for the non-existence of overstability are $Ep_1 > E'q_1$, $Ep_1 > E''q_2$ and $Ep_1 > p_2$.

Proof: Equating the real and imaginary parts of equation (26) and eliminating R_1 between them, we obtain

$$A_3c_1^3+A_2c_1^2+A_1c_1+A_0=0, \tag{43}$$

Here, $c_1=\sigma_1^2$, $b=1+\omega$,

$$A_3 = E'^2 E''^2 q_1^2 p_2^2 q_2^2 \left[\left(\frac{b}{\varepsilon} + \frac{Ep_1}{P} \right) b + \frac{EF_1 p_1 b^2}{P} \right] (b-1), \quad (44)$$

and $$A_0 = \left[\left(\frac{b^8}{\varepsilon} + \frac{Ep_1 b^7}{P} + \frac{EF_1 p_1 b^8}{P} \right) + Q_1 (Ep_1 - p_2) b^6 + \left(S_1 (Ep_1 - E'q_1) + S_2 (Ep_1 - E''q_2) \right) b^5 (b-1) \right] (b-1). \quad (45)$$

The coefficients A_2 and A_1 being quite lengthy and not needed in the discussion of over stability, have not been written here.

Since σ_1 is real for over stability i.e. the three values of $c_1 (= \sigma_1^2)$ should be positive. The product of roots of equation (43) is $-\frac{A_0}{A_3}$, and if this is to be negative, then A_3 and A_0 are of the same sign. Now the product of roots is negative if

$$Ep_1 > E'q_1, Ep_1 > E''q_2 \text{ and } Ep_1 > p_2. \quad (46)$$

Thus, if conditions (46) are satisfied, over stability is impossible and the principle of exchange of stabilities holds true. Therefore, equations (46) are the sufficient conditions for the non-existence of over stability, the violation of which does not necessary involve occurrence of over stability.

6. NUMERICAL RESULTS AND DISCUSSION

For the stationary convection, critical thermal Rayleigh number for the onset of instability is determined for critical wave number. In Fig. 1, critical Rayleigh number R_1 is plotted against solute gradient parameter S_3 for fixed values of $S_2 = 20, Q_1 = 50, F = 10$ and $P = 0.1, 0.3, 0.5$. The critical Rayleigh number increases with increase in solute gradient parameter, which shows that solute gradient has stabilizing effect on the system. In Fig. 2, variation of critical Rayleigh number R_1 is shown against solute gradient parameter S_3 for fixed values of $S_2 = 20, Q_1 = 50, P = 0.3$ and $F = 3, 5, 8$. The critical Rayleigh number increases with increase in solute gradient parameter, which shows that solute gradient has stabilizing effect on the system. In Fig. 3, variation of critical Rayleigh number R_1 is shown against solute gradient parameter S_2 for fixed values of $Q_1 = 50, P = 0.3, F = 5$ and $S_3 = 10, 30, 50$. The critical Rayleigh number increases with increase in solute gradient parameter, which shows that solute gradient has stabilizing effect on the system. In Fig. 4, critical Rayleigh number R_1 is plotted against couple-stress parameter F for fixed value of $S_2 = 20, S_3 = 30, P = 0.3$ and $Q_1 = 20, 30, 40$. The critical Rayleigh number increases with increase in couple-stress parameter, which shows that couple-stress has stabilizing effect on the system. In Fig. 5, critical Rayleigh number R_1 is plotted against couple-stress parameter F for fixed value of $S_2 = 20, S_3 = 30, Q_1 = 30$ and $P = 0.2, 0.4, 0.6$. The critical Rayleigh number increases with increase in couple-stress parameter, which shows that couple-stress has stabilizing effect on the system. In Fig. 6, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $S_2 = 20, S_3 = 30, Q_1 = 20$ and $F = 2, 5, 8$. The critical Rayleigh number decreases with increase in medium permeability, which shows that medium permeability has destabilizing effect on the system. In Fig. 7, critical Rayleigh number R_1 is plotted against medium permeability for fixed value of $S_2 = 20, S_3 = 30, F = 3$ and $Q_1 = 10, 40, 70$. The critical Rayleigh number decreases with increase in medium permeability, which shows that medium permeability has destabilizing effect on the system. In Fig. 8, critical Rayleigh number R_1 is plotted against magnetic field Q_1 for fixed value of $S_2 = 20, S_3 = 30, F = 3$ and $P = 0.2, 0.4, 0.6$. The critical Rayleigh number increases with increase in magnetic field, which shows that magnetic field has stabilizing effect on the system. In Fig. 9, critical Rayleigh number R_1 is plotted against magnetic field Q_1 for fixed value of $S_2 = 20, S_3 = 30, P = 0.3$ and

$F = 2, 5, 8$. The critical Rayleigh number increases with increase in magnetic field, which shows that magnetic field has stabilizing effect on the system.

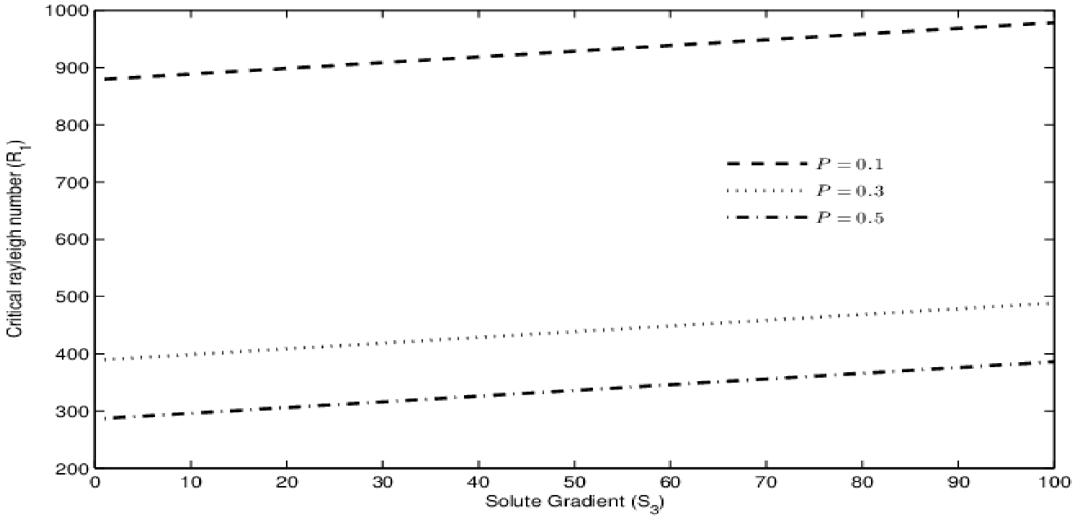

Fig.1 Variations of critical Rayleigh number with solute gradient

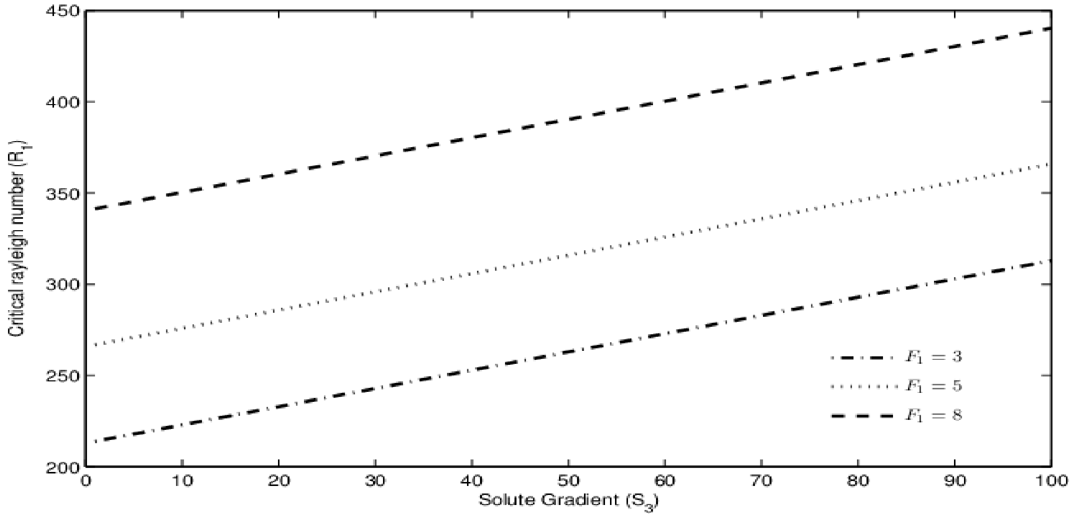

Fig.2 Variations of critical Rayleigh number with solute gradient

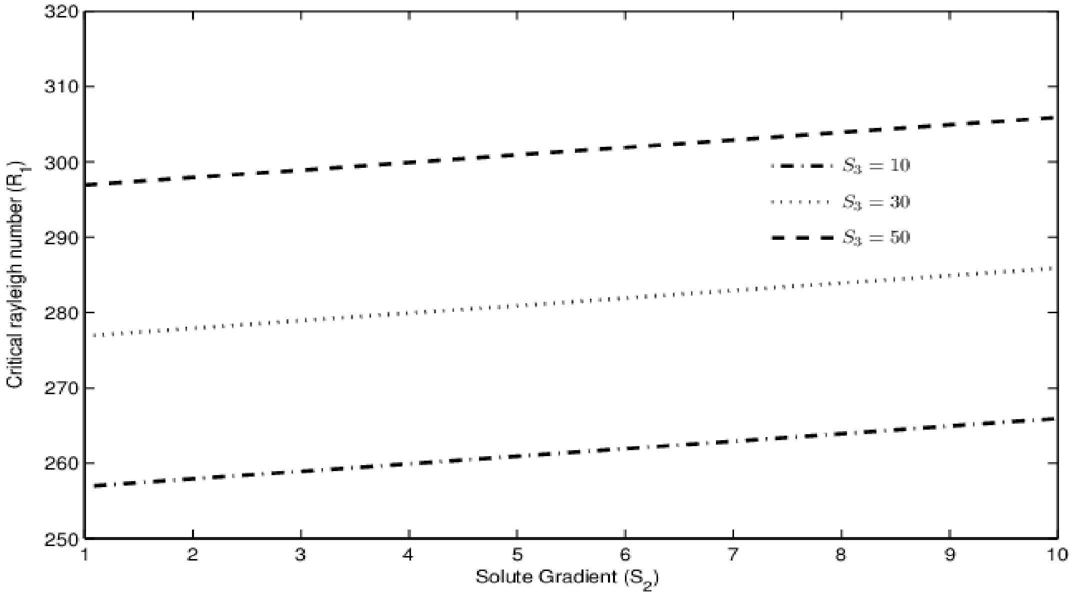

Fig.3 Variations of critical Rayleigh number with solute gradient

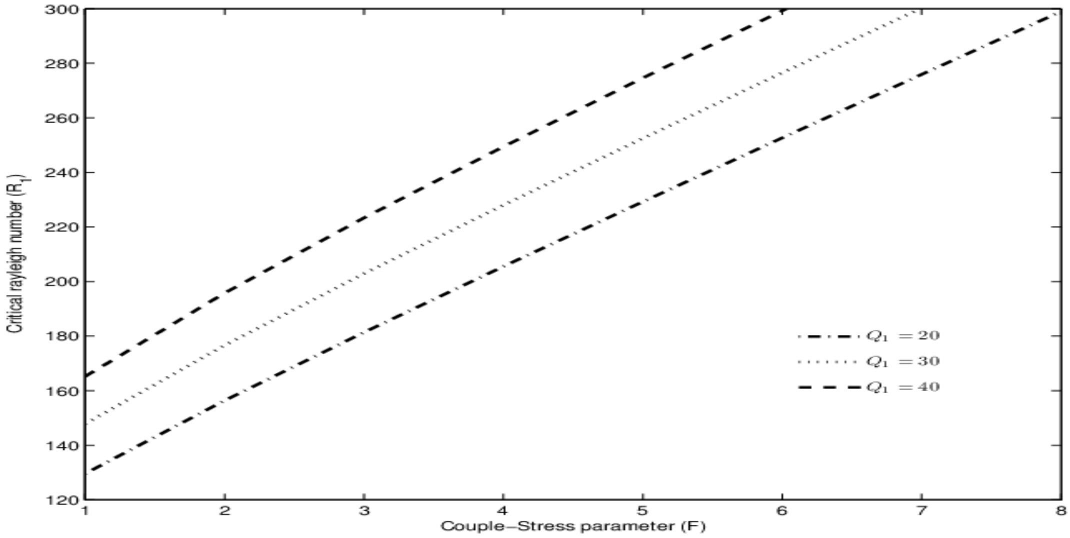

Fig.4 Variations of critical Rayleigh number with couple-stress parameter

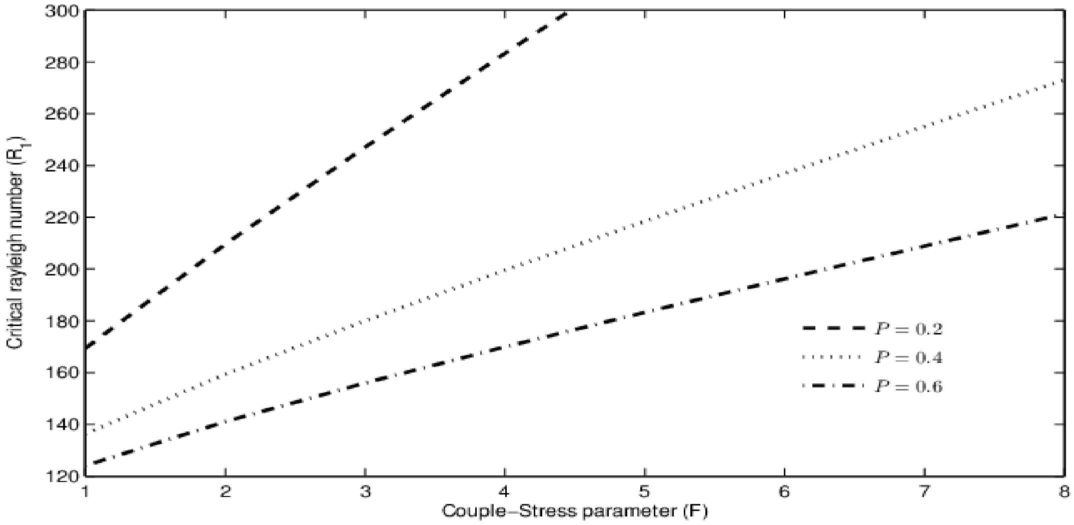

Fig.5 Variations of critical Rayleigh number with couple-stress parameter

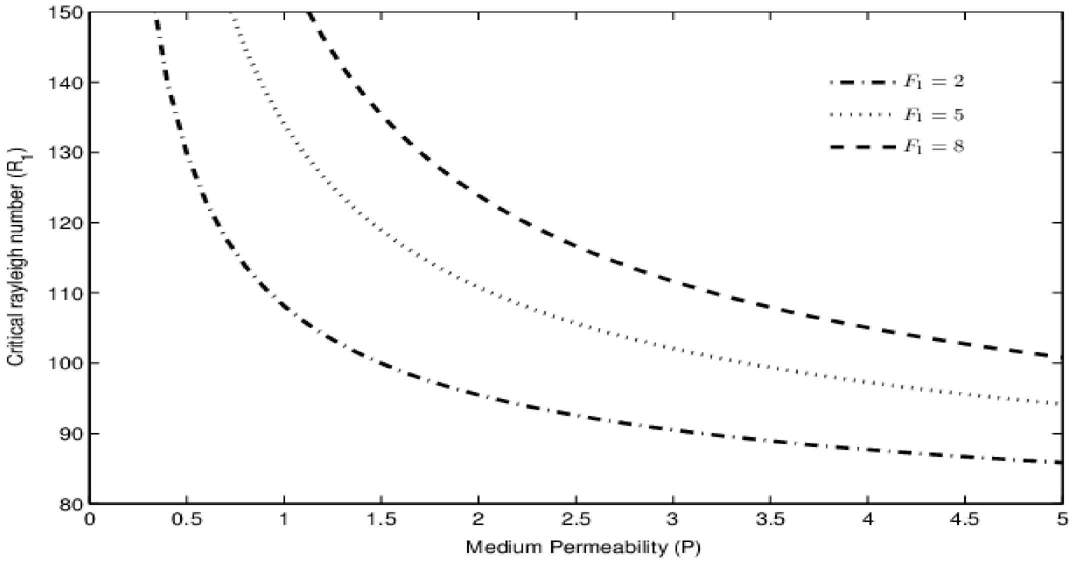

Fig.6 Variations of critical Rayleigh number with medium permeability

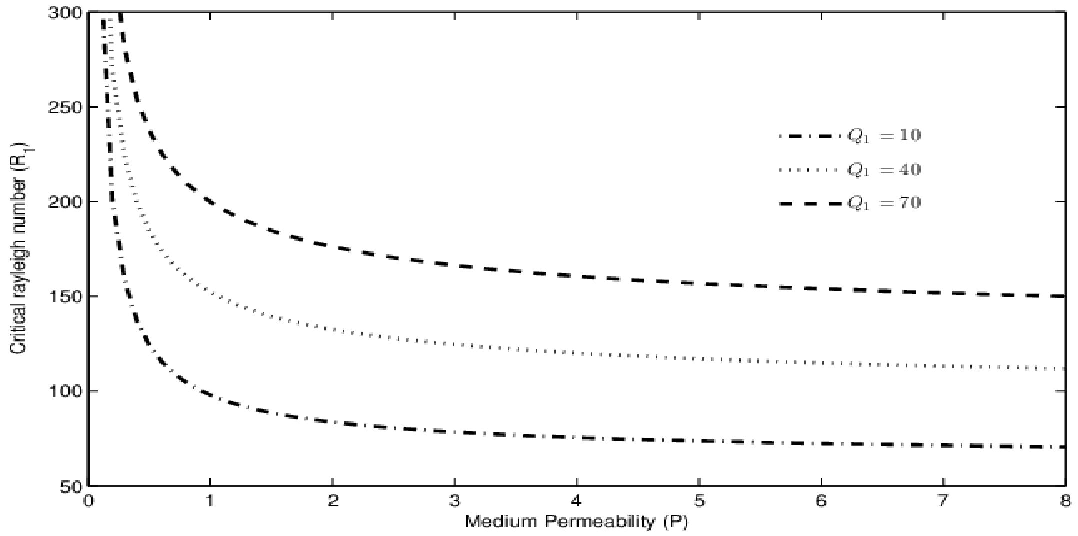

Fig.7 Variations of critical Rayleigh number with medium permeability

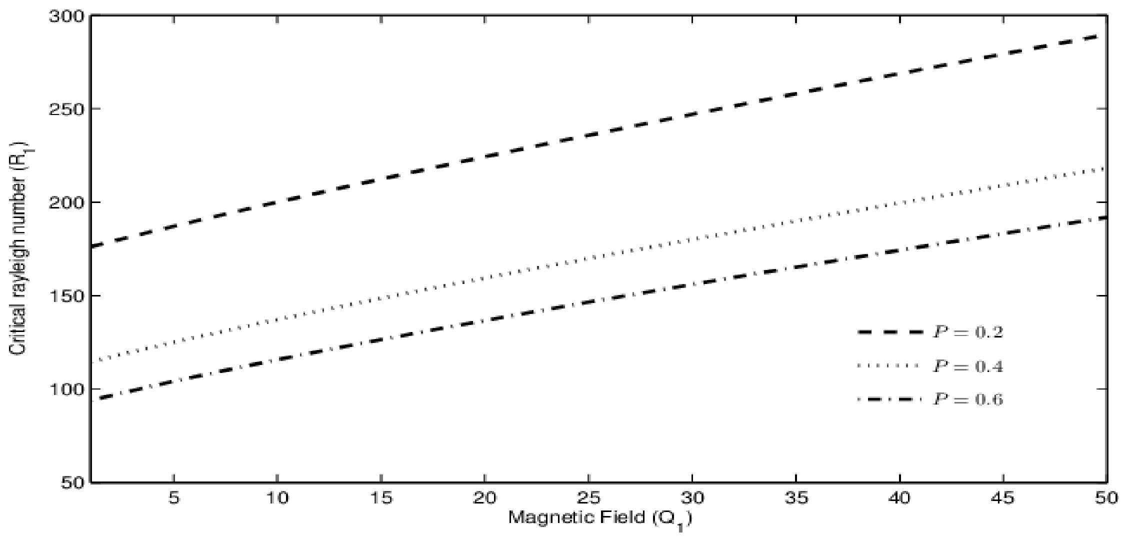

Fig.8 Variations of critical Rayleigh number with magnetic field

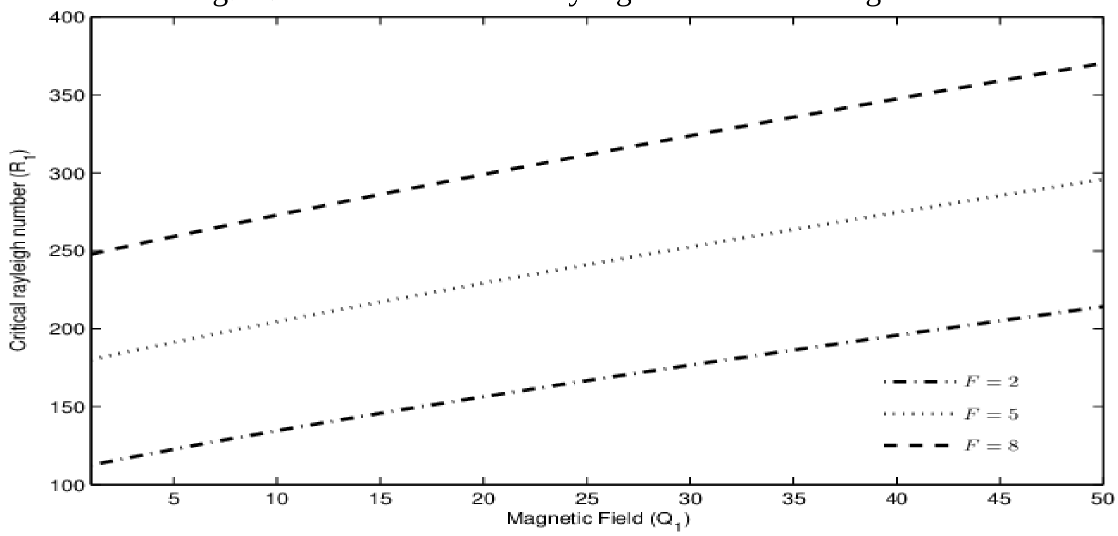

Fig.8 Variations of critical Rayleigh number with magnetic field

7. CONCLUSION

The subject of double-diffusive convection is still an active research area, however, there are many fluid dynamical systems occurring in nature and industrial applications involve three or more stratifying agencies having different molecular diffusivities. More complicated systems can be found in magmas and molten metals. This has prompted researchers to study convective instability in triply diffusive fluid systems. Motivated by this, the effect of uniform vertical magnetic field on triply diffusive convection in a layer of couple-stress fluid heated and soluted from below was considered in the present paper.

The main conclusions from the analysis of this paper are as follows:

(a) For the case of stationary convection the magnetic field, couple-stress parameter and solute gradients have stabilizing effect, whereas the medium permeability has destabilizing effect on the system.

(b) It is observed that stable solute gradients, couple-stress parameter and magnetic field introduce oscillatory modes in the system, which was non-existent in their absence.

(c) In the absence of stable solute gradients, couple-stress parameter and magnetic field, oscillatory modes are not allowed and the principle of exchange of stabilities is valid.

(d) It is found that if $\frac{g\alpha\chi}{\nu\beta}\frac{P_l}{F} \le \frac{27\pi^4}{4}$, the system is stable and under the condition $\frac{g\alpha\chi}{\nu\beta}\frac{P_l}{F} > \frac{27\pi^4}{4}$, the system becomes unstable.

(e) The sufficient conditions for the non-existence of overstability are

$$Ep_1 > E'q_1,\ Ep_1 > E''q_2 \text{ and } Ep_1 > p_2$$

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